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# Interaction of electrons with polaritons

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**Abstract.** The electromagnetic (EM) waves associated with the polariton modes of bulk dielectrics are quantized using a procedure that accounts for both the EM and matter fields. The interaction of electrons with these polariton fields is described using the minimal coupling $(e/m^*)\hat{\boldsymbol{A}}\cdot\boldsymbol{p}$ interaction Hamiltonian where $\hat{\boldsymbol{A}}$ is the transverse vector potential operator of the polariton field, $\boldsymbol{p}$ the electronic momentum and $m^*$ its effective mass. The electron–polariton interaction in the bulk is demonstrated to be closely linked to the behaviour of the polariton group and phase velocities. The same quantization procedure is then employed to describe the EM fields associated with the interface polaritons of a GaAs/AlAs quantum well system. The coupling leads to a dependence of the scattering rate on the group and phase velocities of the surface modes, just as it does for the bulk excitations. In contrast to the case in the bulk, it is shown that these modes *are* important for relaxing the electron energy in narrow wells.

## 1. Introduction

The coupling of electromagnetic (EM) waves with dipole active excitations in dielectrics leads to dressed states known as polaritons. The properties of these normal modes are now well understood. Mills and Burstein [1], Ushioda and Loudon [2], and Cottam and Tilley [3] present a thorough review of their properties both in the bulk and in the presence of surfaces.

The quantization of polaritons was first described by Hopfield [4]. The method has now become standard and involves a Bogoliubov transformation from the normal coordinates of the EM and matter fields to those of the dressed states. Recently, Huttner *et al* [5] have re-examined the Hopfield model from a canonical perspective which resulted in an interesting sum-rule involving the group and phase velocities of the polaritons. Their results are closely related to those of Blow *et al* [6]. These authors, however, were primarily concerned with the quantum optical applications of the modes. Field quantization in dispersive media is currently an important area of investigation in quantum optics that will inevitably overlap with condensed matter physics. In this paper, the motivation lies in understanding the role played by the polaritons in relaxing the energy of electrons both in bulk and low-dimensional systems.

The paper is organized as follows. In section 2 a simple quantization procedure for polaritons in the bulk, starting from the field Hamiltonian for an EM wave in a dispersive dielectric, is outlined. This procedure is shown to be equivalent to the Hopfield model and leads to the sum-rule recently obtained [5]. The interaction of electrons with these bulk modes is discussed first and its significance is assessed for the typical scattering wavevectors encountered in transport phenomena. The formulation for the bulk is however needed, not just for its intrinsic value in obtaining quantized fields in matter, but also in demonstrating the importance of the roles played by the asymptotic properties of the group and phase velocity

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of the excitations. Section 3 extends the above formalisms to a single-quantum-well system (GaAs/AlAs), and specifically, to the interface polaritons. The interaction of electrons with these interface modes is then described quantitatively. The same considerations that apply to the bulk modes lead to very different conclusions when applied to the interface modes. In fact, for small well widths these modes dominate the electron energy relaxation rate. This conclusion is again based on the behaviour of the group and phase velocities together with the dielectric function at the wavevectors of interest. Section 4 contains the conclusions and comments.

## 2. Quantization of bulk polaritons

The quantization of polaritons is briefly dsecribed in this section together with an outline account of their coupling to electrons. The quantization procedure starts by considering an EM field in a simple dispersive dielectric. The field Hamiltonian, which may be obtained from general considerations, is given by the following spectral sum [7]

$$H_f = \frac{1}{2}\epsilon_0 \sum_{\omega_j} \int \left[ \left( \frac{\partial[\omega\epsilon(\omega)]}{\partial\omega} \right)_{\omega=\omega_j} \boldsymbol{E}_j^2 + c^2 \boldsymbol{B}_j^2 \right] \mathrm{d}^3 r. \tag{1}$$

In the above $\boldsymbol{E}_j$ is the electric field, $\boldsymbol{B}_j$ the magnetic field, $c$ the velocity of light *in vacuo*, $\epsilon(\omega)$ the frequency dependent dielectric function (assumed isotropic and real) and $\epsilon_0$ the permittivity of free space. The sum is over all the polariton modes. Their dispersion relation is given by [1]

$$\omega_j^2 \epsilon(\omega_j) = c^2 k^2 \tag{2}$$

with $\boldsymbol{k}$ the mode wavevector.

The electric field operator associated with the $j$th polariton branch is written in quantized form viz.

$$\hat{\boldsymbol{E}}_j(\boldsymbol{r}) = \int [\boldsymbol{E}_{0j}(\boldsymbol{k}) \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}) a_j(\boldsymbol{k}) + \mathrm{HC}]\, \mathrm{d}^3 k \tag{3}$$

where the Boson operators satisfy the usual commutation relations

$$[a_j(\boldsymbol{k}), a_{j'}^{\dagger}(\boldsymbol{k}')] = \delta_{jj'}\delta(\boldsymbol{k}-\boldsymbol{k}') \tag{4}$$

and $\boldsymbol{E}_{0j}(\boldsymbol{k})$ are mode amplitudes to be determined via the canonical procedure. The magnetic field operator may simply be written down as [8]

$$\nabla \times \hat{\boldsymbol{E}}_j(\boldsymbol{r}) = -(\partial/\partial t)\hat{\boldsymbol{B}}_j(\boldsymbol{r}). \tag{5}$$

On substitution of (3) and (5) into (1) and expressing the field Hamiltonian in canonical form

$$\hat{H}_f = \frac{1}{2} \sum_{\omega_j} \int \hbar\omega_j [a_j^{\dagger}(\boldsymbol{k}) a_j(\boldsymbol{k}) + \mathrm{HC}]\, \mathrm{d}^3 k \tag{6}$$

the mode amplitudes reduce to

$$\boldsymbol{E}_{0j}(\boldsymbol{k}) = \{[\hbar\omega_j/2(2\pi)^3\epsilon_0\epsilon(\omega_j)]v_g^{(j)}/v_p^{(j)}\}^{1/2}e_k^{(j)} \tag{7}$$

where $e_k^{(j)}$ is a unit vector in which the label $j$ designates both the branch and the mode polarization; $v_g^{(j)}$ and $v_p^{(j)}$ the group and phase velocities of the mode. To obtain the above, the following identity is employed:

$$1 + [\omega_j/2\epsilon(\omega_j)][\partial\epsilon(\omega)/\partial\omega]_{\omega=\omega_j} = v_p^{(j)}/v_g^{(j)}. \tag{8}$$

The amplitudes given in equation (7) are the three-dimensional analogues of those obtained by Blow *et al* [6] for the special case of one dimension, and are derived here for the first time. Furthermore, using the quantized field in equation (7), the following commutation relation can be established (after some algebra) between the vector potential $\hat{\boldsymbol{A}}(\boldsymbol{r})$ and the electric displacement field $\hat{\boldsymbol{D}}_j(\boldsymbol{r})$ associated with the field. We have

$$[\hat{A}_i(\boldsymbol{r}), -\hat{D}_{i'}(\boldsymbol{r}')] = i\hbar\delta_\perp^{ii'}(\boldsymbol{r}-\boldsymbol{r}')\sum_j \frac{v_g^{(j)}}{v_p^{(j)}} \tag{9}$$

where $\hat{\boldsymbol{E}}_j(\boldsymbol{r}) = -(\partial/\partial t)\hat{\boldsymbol{A}}_j(\boldsymbol{r})$ and $\hat{\boldsymbol{D}}_j(\boldsymbol{r}) = \epsilon_0\epsilon(\omega_j)\hat{\boldsymbol{E}}_j(\boldsymbol{r})$. The sum over the ratio of the velocities is unity [5] demonstrating that $\hat{\boldsymbol{A}}(\boldsymbol{r})$ and $-\hat{\boldsymbol{D}}(\boldsymbol{r})$ are conjugate variables in analogy with the vacuum situation (in the Coulomb gauge). In (9) $\delta_\perp^{ii'}(\boldsymbol{r}-\boldsymbol{r}')$ is the transverse delta-function [9] involving the Cartesian coordinates $i$ and $i'$. The emergence of the group and phase velocities in this manner has important consequences as regards their interaction with electrons.

The interaction of electrons with these polariton modes is now described. As a specific example, phonon-polaritons are considered, although the arguments apply to other types of polaritons, (e.g. plasmon-polaritons and coupled phonon/plasmon-polaritons), via a suitable redefinition of the dielectric function. The appropriate dielectric function here is

$$\epsilon(\omega) = \epsilon_\infty(\omega^2 - \omega_L^2)/(\omega^2 - \omega_T^2). \tag{10}$$

In the above $\epsilon_\infty$ is the high-frequency dielectric constant with $\omega_L$ and $\omega_T$ the zone-centre LO and TO optical phonon frequencies. The dispersion relation obtained by substituting (10) into (2) is well known [1] and consists of a 'photon-like' upper branch (+) and a 'phonon-like' lower branch (−) where the wavevectors of interest are of order $k_0 = \omega_T\epsilon_\infty^{1/2}/c$.

The interactions of electrons with the EM wave associated with the polariton occurs via the usual minimal coupling $(e/m^*)\hat{\boldsymbol{A}}\cdot\boldsymbol{p}$ Hamiltonian, with $m^*$ the effective mass and $\boldsymbol{p}$ the momentum of the electron. Explicit evaluation leads to the result that the scattering rate $\Gamma_j$, calculated via Fermi's golden rule, is then proportional to

$$\Gamma_j \propto \frac{1}{\omega_j\epsilon(\omega_j)}\frac{v_g^{(j)}}{v_p^{(j)}}. \tag{11}$$

Typical scattering wavevectors are of the order [10] $q = (2m^*\omega_T/\hbar)^{1/2} \simeq 0.4\times 10^3 k_0$; hence for these wavevectors, the scattering from the lower branch is negligible since

$$\frac{v_g^{(-)}}{v_p^{(-)}}\frac{1}{\epsilon(\omega_-)} \to 0 \qquad \text{(within a few } k_0\text{)}. \tag{12}$$

We can also show that the interaction of electrons with the 'photon-like' upper branch is negligible too, but for different reasons. In this case, for typical scattering wavevectors, the ratio of the velocities is unity, $\epsilon(\omega_+) \simeq \epsilon_\infty$ and $\omega_+ \simeq 0.4 \times 10^3 \omega_T$. The argument involving $v_g$ and $v_p$ transparently leads to the conclusion that electron energy relaxation via the emission of *bulk polaritons* may safely be ignored. By contrast as we show below, the asymptopic properties of $v_g$ and $v_p$ are drastically different in the context of interface polaritons leading to significant contribution to the relaxation rates.

## 3. Interaction of electrons with interface polaritons

The formalism developed in the previous section can now be applied to the interface polaritons of a layered system. It is well known that such a system may support a rich spectrum of polariton excitations, both guided and interface modes [2,3]. The guided modes interact with electrons only very weakly for precisely the same reasons as given for the bulk modes in section 2. On the other hand, the interface modes, often referred to in the literature as Fuchs–Kiewer (FK) modes [11] certainly cannot be ignored as is now demonstrated.

As a concrete example, the formalism developed in section 2 will be extended in order to describe the interaction of electrons in a GaAs/AlAs double heterojunction (DH). In what follows labels 1 and 2 refer to AlAs ($|z| > d/2$) and GaAs ($|z| < d/2$) respectively, with $d$ the well width. It is assumed that each region may be described adequately by its bulk dielectric function, viz.

$$\epsilon_i(\omega) = \epsilon_{\infty i}(\omega_{Li}^2 - \omega^2)/(\omega_{Ti}^2 - \omega^2) \qquad i = 1, 2 \tag{13}$$

with $\epsilon_{\infty i}$ the high-frequency dielectric constant, and $\omega_{Li}$ and $\omega_{Ti}$ the zone centre LO and TO phonon frequencies of material $i$. The dispersion relation of these FK interface polaritons is simply obtained by applying standard electromagnetic boundary conditions at the interfaces and seeking decaying solutions on either side of the boundaries. This dispersion relation is expressible as [2,3,11]

$$\frac{\epsilon_2(\omega)q_1}{\epsilon_1(\omega)q_2} = \begin{cases} -\coth(q_2 d/2) & \text{(S)} \\ -\tanh(q_2 d/2) & \text{(A)} \end{cases} \tag{14}$$

where S and A denote symmetric and antisymmetric solutions, the labelling of the modes being consistent with that of other investigations such as those of Wendler [12] and Mori and Ando [13]. In the above the wavevectors $q_i$ are given by

$$q_i^2 = q_\parallel^2 - \omega^2 \epsilon_i(\omega)/c^2 \qquad i = 1, 2 \tag{15}$$

with $q_\parallel$ the wavevector in the plane. For the wavevectors of interest in transport, which are well away from the light-line, it is safe to consider the so-called unretarded limit ($q_i = q_\parallel$) although, initially, the discussion is kept general. Both regions of the dispersion curve are depicted in figure 1.

It is now straightforward to quantize these modes. The electric field is written in quantized form

$$\hat{E}_j(r) = \int [E_{0j}(q_\parallel) \exp(\mathrm{i} q_\parallel \cdot r_\parallel) a_j(q_\parallel) + \mathrm{HC}]\, \mathrm{d}^2 q_\parallel \tag{16}$$

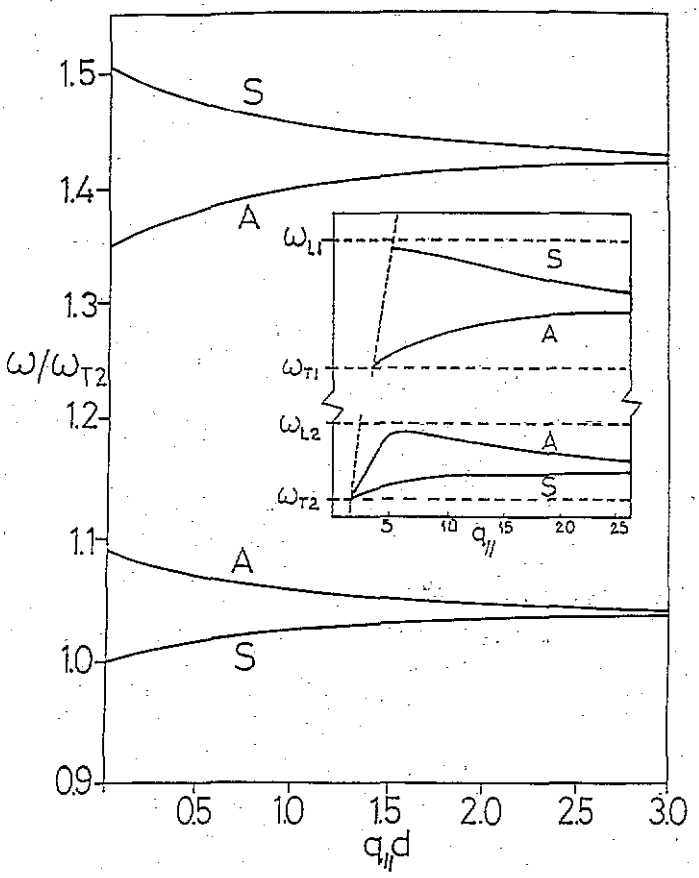


**Figure 1.** The four FK interface polariton branches in a GaAs/AlAs DH. The relevant parameters are $\hbar\omega_{L1} = 50.09$ meV, $\hbar\omega_{T1} = 44.88$ meV, $\hbar\omega_{L2} = 36.25$ meV, $\hbar\omega_{T2} = 33.29$ meV, $\epsilon_{\infty 1} = 8.16$, $\epsilon_{\infty 2} = 10.89$ where 1 (2) refers to AlAs (GaAs). The inset to the figure shows the small $q_\parallel$ region of the dispersion curves in the vicinity of the light-line for $d = 1$ $\mu$m. The horizontal scale is in units of $\omega_{T2}/c$. The labels S (A) refer to the symmetric (antisymmetric) modes.

where, of course, now we sum over the in-plane wavevector $q_\parallel$ which is the conserved wavevector by symmetry, and $r_\parallel$ is the in-plane coordinate. The index $j$ now refers to either the antisymmetric (A) or symmetric (S) branch. The magnetic field operator is obtainable from (16) using (5) and hence the total energy of the modes can again be determined from (1). On expressing this in canonical form, the mode amplitudes $E_{0j}$ are evaluated. The concern here is with the electron–interface polariton interaction, so only the vector potential $\hat{\boldsymbol{A}}(\boldsymbol{r})$ is quoted:

$$\hat{\boldsymbol{A}}(\boldsymbol{r}) = -\mathrm{i}\sum_j \int [\boldsymbol{A}_{0j}(\boldsymbol{q}_\parallel)\exp(\mathrm{i}\boldsymbol{q}_\parallel \cdot \boldsymbol{r}_\parallel)a_j(\boldsymbol{q}_\parallel) - \mathrm{HC}]\,\mathrm{d}^2\boldsymbol{q}_\parallel. \tag{17}$$

In equation (17) the amplitude is given by

$$\boldsymbol{A}_{0j} = \left\{\frac{\hbar\omega_j}{(2\pi)^2\epsilon_0 D_E^{(j)}}\right\}^{1/2} \{\hat{\boldsymbol{r}}_\parallel F_j(z) + \hat{\boldsymbol{z}}G_j(z)\} \tag{18}$$

where the carets denote unit vectors. The structure factors $F_j(z)$ and $G_j(z)$ are written in the following concise form:

$$F_S(z) = [\exp(q_1 z)|a\cosh(q_2 z)/\cosh(q_2 d/2)|\exp(-q_1 z)] \tag{19}$$

$$F_A(z) = [-\exp(q_1 z)|a\sinh(q_2 z)/\sinh(q_2 d/2)|\exp(-q_1 z)] \tag{20}$$

$$G_S(z) = (-iq_\parallel/q_1)[\exp(q_1 z)|aq_1\sinh(q_2 z)/q_2\cosh(q_2 d/2)| - \exp(-q_1 z)] \tag{21}$$

$$G_A(z) = (-iq_\parallel/q_1)[-\exp(q_1 z)|aq_1\cosh(q_2 z)/q_2\sinh(q_2 d/2)| - \exp(-q_1 z)] \tag{22}$$

with $a = \exp(-q_1 d/2)$ and the vertical bars represent the left boundary (at $z = -d/2$) and the right boundary (at $z = d/2$) of the DH. $D_E^{(j)}$ is the energy normalization factor originating from the $E^2$ term in (1) and is given by

$$D_E^{(j)} = -g_j \frac{\omega_j^2 \exp(-q_\parallel d)d\epsilon_1^2(\omega_j)}{\epsilon_2(\omega_j)} \frac{v_p^{(j)}}{v_g^{(j)}} \begin{cases} \operatorname{cosech}^2(q_\parallel d/2) & j = S \\ \operatorname{sech}^2(q_\parallel d/2) & j = A. \end{cases} \tag{23}$$

In equation (23), $g_j$ is 1 if $j = S$ and $-1$ if $j = A$. The group and phase velocities are of course defined for the surface polaritons in terms of the in-plane wavevector ($v_p^{(j)} = \omega_j/q_\parallel$ and $v_g^{(j)} = [\partial\omega/\partial q_\parallel]_{\omega=\omega_j}$). The group velocity of the modes can be determined analytically from the dispersion relation and are found to be given by

$$v_g^{(j)} = \frac{d\epsilon_1^2(\omega_j)}{2\Xi(\omega_j)} \begin{cases} \operatorname{cosech}^2(q_\parallel d/2) & j = S \\ -\operatorname{sech}^2(q_\parallel d/2) & j = A \end{cases} \tag{24}$$

with

$$\Xi(\omega) = \epsilon_1(\omega)\frac{\partial\epsilon_2(\omega)}{\partial\omega} - \epsilon_2(\omega)\frac{\partial\epsilon_1(\omega)}{\partial\omega}. \tag{25}$$

In figure 2 the ratio $|v_g/v_p|$ is plotted as a function of the in-plane wavevector. It is seen from the figure that the dependence of this ratio on wavevector is not too different for the various interface modes. At large wavevectors the ratios fall off rather slowly; on the other hand, for small wavevectors, there is a rapid fall-off. For intermediate values, these ratios attain a maximum around $q_\parallel d = 1$, which is often near the wavevector regime of interest in the electron–interface mode interaction.

The analysis is now taken further by calculating the electron–interface mode interaction (some results have recently been presented elsewhere [14]). The rates are calculated via the golden rule (assuming that only emission is possible):

$$\Gamma_j(k_\parallel^i) = \frac{2\pi}{\hbar}\int d^2q_\parallel \int d^2k_\parallel^f |\langle k_\parallel^f\{q_\parallel j\}|\left(\frac{e}{m^*}\right)A^{(j)}\cdot p|k_\parallel^i\{0\}\rangle|^2\delta(E_i - E_f - \hbar\omega_j) \tag{26}$$

with i and f standing for the initial and final states. The ket $|k_\parallel\rangle$ represents that of an electron with in-plane momentum $\hbar k_\parallel$, whilst the ket $|\{q_\parallel j\}\rangle$ describes a polariton of momentum $\hbar q_\parallel$ of branch $j$. The electron states are assumed for simplicity to be those obtained from an infinite confining potential, although for the low-lying states this assumption is not too severe. Hence, on simple symmetry considerations, intersubband transitions will only involve the antisymmetric FK modes whereas only symmetric FK modes contribute to

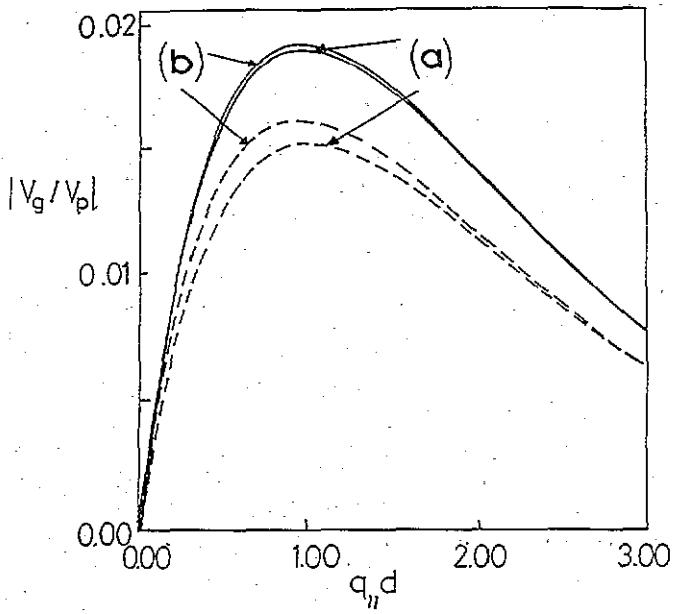


**Figure 2.** Variation of $|v_g/v_p|$ with $q_\parallel d$ for the symmetric FK modes (curves a) and the antisymmetric FK modes (curves b). The full curves correspond to the 'AlAs-like' modes and the broken curves to the 'GaAs-like' modes.

intrasubband scattering. For intersubband scattering where the electron initially resides at the bottom of the second subband ($k_\parallel^{(i)} = 0$) the integrations involved in (26) are straightforward leading to

$$\frac{\Gamma_A}{\Gamma_0} = (4\pi)^3 \cosh^2(q_\parallel d/2) \coth^2(q_\parallel d/2) \left[\frac{E_1}{\hbar\omega_A}\right] \left(\frac{\hbar}{2m_2^*\omega_{L2}d^2}\right)^{1/2} \times \left(\frac{\epsilon_{s2}\epsilon_{\infty 2}}{\epsilon_{s2}-\epsilon_{\infty 2}}\right) \left[\frac{1}{\pi^2+q_\parallel^2 d^2} - \frac{3}{9\pi^2+q_\parallel^2 d^2}\right]^2 \left|\frac{\epsilon_2(\omega_A)}{\epsilon_1^2(\omega_A)} \frac{v_g^{(A)}}{v_p^{(A)}}\right| \tag{27}$$

with $E_1 = \hbar^2\pi^2/2m_2^*d^2$. The quantity $\Gamma_0$ characterizes the strength of the electron–LO phonon interaction in the bulk ($\sim 8 \times 10^{12}$ s$^{-1}$ for GaAs):

$$\Gamma_0 = (e^2/4\pi\epsilon_0\hbar)(1/\epsilon_{\infty 2} - 1/\epsilon_{s2})(2m_2^*\omega_{L2}/\hbar)^{1/2}. \tag{28}$$

Of course, $q_\parallel$ in (27) satisfies both the dispersion relation and conservation of energy and there are two branches which may contribute: an 'AlAs-like' antisymmetric branch and a 'GaAs-like' antisymmetric branch (see figure 1). In figure 3 the intersubband rate is depicted as a function of well width. The 'AlAs-like' contribution is seen to dominate for all well thicknesses $d$, especially around 180 Å where a pronounced resonance is predicted [14]. The reason for this is easily seen with reference to equation (27) as follows. As the well width increases, the rate increases due to a decrease in $q_\parallel$. As the in-plane wavevector decreases further, the frequency of the 'AlAs-like' antisymmetric branch tends to $\omega_{T1}$, whence $|\epsilon_1(\omega)| \to \infty$ reversing the earlier trend. In other words, when the mode frequency approaches the TO phonon frequency of AlAs, the rate vanishes, as indeed it should. It

should be emphasized that this resonance in the intersubband rate is not due to retardation effects, but rather to the dispersion of the mode, or more precisely, to its finite group velocity in the non-retarded region of the dispersion curve (figure 1). This resonance is absent if dispersion is ignored. No such resonance is predicted for the 'GaAs-like' contribution as here the limiting frequency is that of the LO mode of GaAs. The intersubband rate for the 'GaAs-like' antisymmetric mode increases until the subband separation equals the energy of the GaAs LO phonon (corresponding to a well width of around 220 Å). For larger well widths, the electron has insufficient energy to emit these modes.

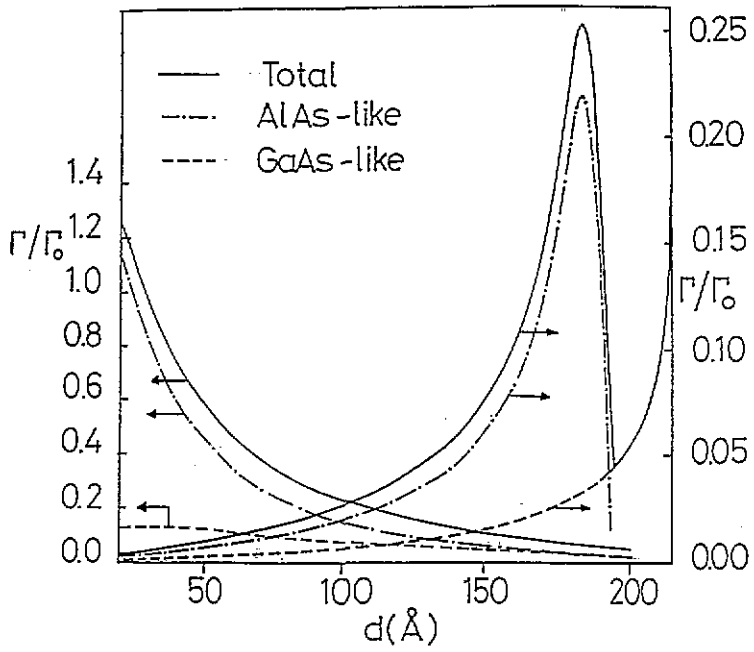


**Figure 3.** Intra- (left-hand axis) and inter- (right-hand axis) subband transition rates by emission of FK antisymmetric modes and their variation with well width $d$.

No analytic results are available for the intrasubband rates with the integrations involved in (26) having to be evaluated numerically. Figure 3 also illustrates the variation of the intrasubband scattering rate as a function of well width for a fixed initial electron energy of $2\hbar\omega_{T2}$. Again the 'AlAs-like' symmetric branch dominates, especially at small well widths. This result has recently been demonstrated by the Raman results of Tsen *et al* [15]. In order to make a quantitative comparison with these results the theory needs to be taken further, for example by taking into account the finite depth of the well, and, more importantly, effects due to non-equilibrium phonons.

## 4. Discussion and conclusions

The aim of this paper has been to demonstrate that the interactions of electrons with the polariton modes can be presented in a unifying manner irrespective of whether they occur in bulk or lower-dimensional systems. For bulk systems, the interaction of electrons with

these modes occurs via the $(e/m^*)\hat{A}\cdot p$ minimal coupling Hamiltonian, but the strengths are so weak that they may safely be ignored. The reason for this, as explained in section 2, is intimately related to the group and phase velocities of the modes and the behaviour of the dielectric function at the wavevector regimes encountered in transport. The interaction of electrons with interface FK modes employed here was again $(e/m^*)\hat{A}\cdot p$, in order to connect with the bulk results of section 2. Other investigators, for example Wendler [12] and Mori and Ando [13] couple the modes via a scalar potential coupling $e\hat{\phi}$ analogous to the Fröhlich interaction of the LO modes. The use of this type of interaction for polaritons has recently been questioned [16, 17]. The results obtained by this scalar potential method are equivalent to those reported here and a simple argument will demonstrate this.

In the non-retarded limit the vector potential $\hat{A}$ may be written as

$$\hat{A} \simeq (\mathrm{i}/\omega)\nabla\hat{\phi} = -(k/\omega)\hat{\phi} \tag{29}$$

hence

$$(e/m^*)p\cdot\hat{A} \simeq (\mathrm{i}e/\omega)\hat{\phi}k\cdot v_e \tag{30}$$

where $v_e$ is the group velocity of the electron and $k$ the wavevector of the mode. For the interactions of interest $k\cdot v_e$ is the frequency of the emitted mode and hence the $e\hat{\phi}$ coupling is recovered. Nevertheless, the essence of their interactions with electrons stems from the properties of their group velocities, their phase velocities, and the dielectric functions of the media, just as it does for bulk polaritons. No such comparison can be made between these FK interface modes and bulk LO phonons. The interaction of electrons with bulk and confined LO phonons via the Fröhlich interaction is of course strong, but it cannot be for exactly the same reasons. In the case of the long-wavelength LO modes $\epsilon(\omega) = 0$, and their group velocity also vanishes (in the often used Einstein model). A simple derivation of the Fröhlich interaction for longitudinal modes with equation (1) as the starting point has recently been reported by one of the present authors [18].

In conclusion, a simple quantization procedure for polaritons has been presented which is applicable to any system in which a simple dielectric function is adequate. The group velocity is shown to be of importance from the outset, and the commutation relation (9) recently obtained by Huttner *et al* [5] is recovered. The interaction of electrons with bulk polaritons is shown to be negligible from simple considerations. These same considerations when applied to layered systems lead to the result that the fields associated with surface polaritons cannot be ignored in any calculations of electron energy relaxation, despite the fact that their coupling to electrons has precisely the same interaction Hamiltonian as that of bulk polariton modes. This is a consequence of the behaviour of the group and phase velocities and the dielectric functions at the large wavevectors of interest.

A note of caution must now be made. Microscopic models [19–22] and macroscopic ones [23–26] demonstrate that the 'GaAs-like' interface mode is not a pure interface mode but is strongly hybridized with the LO modes. This is not the case for the important 'AlAs-like' mode [19]. The reason for this is that the dispersion of the LO phonons in AlAs is too weak to allow hybridization over the whole frequency range of the interface modes. Hybrids occur only in the frequency range occupied by LO modes. Hybrid behaviour has been ignored here as it has been in other investigations [12, 13] although Ridley [24] has recently calculated electron–optical phonon scattering rates for hybrid modes with frequencies in the 'GaAs-like' region of the dispersion curves. As has recently been emphasized [26] a macroscopic model which incorporates hybrid effects and accurately compares with microscopic displacements and frequencies is useful in order to avoid heavy computation.

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*Note added in proof.* The equivalence in the non-retarded limit of the $\hat{A}$ and $\hat{\phi}$ formalisms discussed in section 4 has recently been demonstrated more rigorously via a unitary transformation (M Babiker, N C Constantinou and B K Ridley *Phys. Rev.* B at press).

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